Evolutionary Bi-Objective Optimization and Knowledge Extraction for Electronic and Automotive Cooling

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Abstract. The heat sink is one of the most widely used devices for thermal management of electronic devices and automotive systems. The present study approaches the design of the heat sink with the aim of enhancing their efficiency and keeping the material cost to a minimum. The above-mentioned purpose is achieved by posing the heat sink design problem as a bi-objective optimization problem where entropy generation rate and material cost are the two conflicting objective functions. The minimum entropy generation rate reduces irreversibilities inherent in the system, thus leading to improved performance, while the reduction in material cost ensures its economic feasibility. This bi-objective optimization problem is solved using Non-dominated Sorting Genetic Algorithm (NSGA-II) in the presence of geometric restrictions and functional requirements. Heat sinks with two different flow directions, namely flow-through air cooling system and impingement-flow air cooling system, are optimized to identify the best geometric and flow parameters. Subsequently a knowledge extraction exercise is carried out over non-dominated solutions obtained from the multi-objective optimization, to establish a relationship between the objective function and involved design parameters. The knowledge extracted has significant potential to simplify the calculations performed by thermal engineering experts in the selection of the heat sink for a specific application.

Keywords: Evolutionary Computation and Bi-Objective Optimization · Knowledge Extraction · Electronic and Automotive Cooling · Plate-Fin Heat Sink.

1 Introduction

Every electronic device and automotive system need to dissipate a certain amount of heat to maintain its temperature within its operational range. This activity of controlling the heat to be rejected is known as thermal management. One of the most significant ways to achieve thermal management is to reduce thermal resistance. This can be ensured by increasing the surface area of the interfacing surface between heat generating devices and the cooling medium. The simplest way to enhance cooling under cost, space, and weight constraints is to use a heat sink with the fin. Plate-Fin Heat Sinks (PFHS) are generally integrated into electronic devices that cool by blowing-out the heat. The PFHS is in principle a heat exchanger component that cools the device by dissipating heat to the surrounding cooling medium. The PFHS consists of two major parts - one part is a flat plate which is intended to make good thermal contact with the electronic device, and the other part is an array of comb-like protrusions to increase the surface area in contact with the cooling medium.

There have been several efforts to understand, apply, and improve the functioning of PFHSs. In the modern era, when every design has to pass through the philosophy of sustainable development, the design of the heat sink cannot be an exception. Bar-Cohen [2] observed that the sustainable development of the PFHS involves a subtle balance between a superior thermal design, minimum material consumption, and minimum pumping power. Bejan and Morega [4] introduced the concept of the minimization of the entropy generation. Culham and Muzychka [8] presented simultaneous optimization of PFHS design based on minimization of the entropy generation associated with heat transfer and fluid friction. Chen et al. [6] considered the minimization of entropy generation rate to be the objective function which is able to account for air resistance as well as the heat transfer resistance simultaneously. Mohsin et al. [12] applied Genetic Algorithm (GA) to minimize entropy generation rate due to heat transfer and pressure drop across pin fins. Culham et al. [7] highlighted the importance of the contribution made by all thermal resistance elements including contact resistance and spread resistance etc. between the heat source and the sink to the entropy generation. Ndao et al. [13] performed multi-objective [3] thermal design optimization and comparative study of various cooling technologies like continuous parallel micro-channel PFHSs, inline and staggered circular pin-fin PFHSs, offset strip fin PFHSs, and single and multiple submerged impinging jet(s). Mohsin et al. [12] used Genetic Algorithms (GAs) to minimize the entropy generation rate and demonstrated that geometric parameters, material properties, and flow conditions can be simultaneously optimized using GA. Sanaye and Hajabdollahi [14] carried out a thermo-economic optimization of the plate fin heat exchanger using genetic algorithms. A hybrid method was proposed by Ahmadi et al. [1] which is known as Genetic Algorithm Hybrid with Particle Swarm Optimization (GAHPSO) for design optimization of a plate-fin heat exchanger. The algorithm is able to handle both continuous and discrete variables. Another study of plate-fin heat exchangers was proposed by [11] using biogeography-based optimization (BBO). Wang and Li [17] proposed a method to address the problem of decrease in heat transfer performance and increase of pressure drop arising due to inappropriate surface selection and layer pattern. Ventola et al. [16] developed a novel thermal model of the PFHS, validated it experimentally, and demonstrated its superior accuracy.

The principle of the minimum entropy generation rate produces PFHS designs which are not only thermodynamically efficient but also have better geometric and topological features. This paper proposes a multi-objective optimization approach of PFHSs that are used as a cooling mechanism in electronic devices. The different variables of the optimization study are number of fins, height of the fin, spacing between the fins, and incoming air velocity. In addition to the restrictions on the lower and upper bounds of the design variables, there are also a few non-linear constraints from geometrical dependency, design specifications, and functional requirements. Two configurations such as PFHS with a flow-through air cooling system and PFHS with an impingement-flow air cooling system are considered in the present work. The PFHSs with impingement flow are used to obtain high local and area averaged heat transfer coefficients in the convective heat transfer process. Therefore, this configuration is used where heat flux density is significantly high, like cooling of turbine blades. The PFHS with flow through configuration is used when the constraint on the space availability is relatively relaxed. The conflicting objectives of multi-objective optimization are entropy generation rate and cost [9]. Minimum entropy generation rate will ensure better cooling. However, the solutions might not be economical. The minimum cost will ensure a design that works better from the economic point of view.

The structure and scope of the rest of the paper are organized as follows: Section 2 presents the details about the PFHS along with the heat sink design as a multi-objective problem defining design variables, constraints, and objective functions. The optimization results and the corresponding plots are discussed in Section 3. In Section 4, knowledge extraction methodologies are applied to the results obtained from the optimization study to establish a knowledge base for future reference. Finally, the concluding remarks and future development scope of the study are presented in Section 5.

2 Plate-fin Heat Sink (PFHS)

There exists a large number of analysis tools for the determination of the thermal performance of PFHSs, provided design conditions are well defined. A model proposing a relationship between entropy generation and material cost with PFHS design parameters can be optimized in such a manner that relevant design parameters attain a value which combines to produce the best possible PFHS performance for a given set of constraints [15]. Two different configurations of a PFHS are considered in this paper. The first one is the flow-through air cooling system in Fig. 1 and the second one is the impingement-flow air cooling system in Fig. 2. The first flow configuration is used where a relatively large space is available as cooling fluid flows along the PFHS and not directly on the hot surface. The impingement-flow air cooling system (shown in Fig. 2) is suitable for the applications where large electronic component density exists and high heat flux needs to be dissipated. In this cooling arrangement, the goal is achieved by impingement of high velocity cooling fluid directly on the surface to be heated.

2.1 Multi-Objective Optimization Problem Formulation

In the present work, an attempt has been made for simultaneous minimization of two conflicting objectives, the entropy generation rate (thermal performance) and the material cost (economy). This problem is adopted from Chen and Chen [5]. The multi-objective optimization design problem can be formulated as follows:

$$\begin{aligned} C_{mat} &= (w \times L \times t_b + N \times H \times b \times L) \times \rho \times Cost. \\ \dot{S}_{gen} &= R_{sink} \times \left(\frac{\dot{Q}}{T_{amb}}\right)^2 + \frac{F_d \times V_f}{T_{amb}}. \end{aligned} \tag{1}$$

Where C_{mat} is the cost of material from which the PFHS is made, w width of the fin, L length of fins, t_b base length of fins, N number of fins, H height of the fin, b spacing

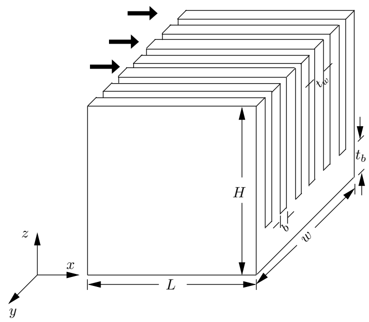


Fig. 1. PFHS with flow-through cooling.

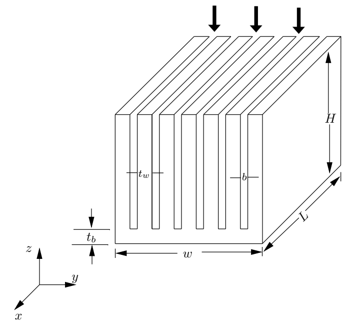


Fig. 2. PFHS with impingement-flow cooling.

between fins, ρ density of the cooling fluid, $\dot{S}_{gen}$ entropy generation rate, R_{sink} overall PFHS thermal resistance, $\dot{Q}$ represents heat generation rate, T_{amb} absolute surrounding temperature, F_d fluid friction in the form of drag force, and V_f uniform stream velocity. The overall PFHS resistance as defined in the case of flow-through air cooling systems and impingement-flow air cooling systems are given by:

$$R_{sink} = \begin{cases} \dfrac{1}{\left(\dfrac{N}{R_{fin}}\right) + (h_{eff} \times (N-1) \times b \times L)} \\ + \dfrac{t_b}{k \times L \times w}, \text{for flow-through air inlet,} \\ \dfrac{1}{h_{eff} \times A \times \eta_{fin}}, \text{for impingement-flow air inlet.} \end{cases} \tag{2}$$

where R_{fin} is the thermal resistance of a single fin, h_{eff} is the effective heat transfer coefficient (the fins being assumed as straight fins with an adiabatic tip), k is the thermal conductivity, A is the total surface area of PFHS and other exposed surfaces, and η_{fin} represents the total heat dissipation efficiency. The objective functions are subjected to the following constraints:

$$\begin{aligned} &g_1 : 0.001 - \left(\frac{w-t_w}{N-1} - t_w\right) \leq 0, \\ &g_2 : \left(\frac{w-t_w}{N-1} - t_w\right) - 0.005 \leq 0, \\ &g_3 : 0.001 - \left(\frac{H}{\left(\dfrac{w-t_w}{N-1}\right) - t_w}\right) \leq 0, \\ &g_4 : \left(\frac{H}{\left(\dfrac{w-t_w}{N-1}\right) - t_w}\right) - 194.0 \leq 0, \\ &g_5 : 0.0001 - \sqrt{\frac{\left(\dfrac{w-t_w}{N-1} - t_w\right) V_{ch}}{\nu} \times \frac{\left(\dfrac{w-t_w}{N-1} - t_w\right)}{L}} \leq 0. \end{aligned} \tag{3}$$

The first two constraints, g_1 and g_2, put a limit on the fin gap, and according to these constraints the fin gap should lie in the range of 0.001 m to 0.005 m. The other two constraints deal with design specifications (g_3 and g_4) that arise due to limited space for installation. According to these constraints, the fin aspect ratio (ratio of height and thickness of the fin) should lie in the range of 0.01 and 194. The constraint g_5 is simply to avoid getting a zero Reynolds number. Beside these constraints, the design parameters can attain only those values which fall in the admissible limits. These admissible values of the design parameters are as follows:

$$\begin{array}{lll} 2 & \leq N & \leq 40, \\ 0.014 & \leq H & \leq 0.025, \\ 2 \times 10^{-4} & \leq b & \leq 2.5 \times 10^{-3}, \\ 0.5 & \leq V_f & \leq 2, \\ & N \times b & \leq 0.05. \end{array} \tag{4}$$

3 Optimization Results

In the present work, an attempt has been made to simultaneously minimize of two conflicting objectives - the entropy generation rate (from the thermal performance perspective) and the material cost (from the economic perspective). It is observed from Eq. 2 that in case of flow through configuration, the thermal resistance of the PFHS is inversely proportional to number of fins. As increase in the number of fins also translates into enhanced exposed surface area, in case of impingement flow also, the inverse relationship between the thermal resistance and the number of fins remains valid. As a result, in both flow configurations increase in number of fins apparently leads to decrease in entropy generation rate ($\dot{S}_{gen}$). However, it is pertinent to note that the increase in number of fins also results in increased drag force being offered to the fluid flow. This increase in drag force has a consequential effect of increase in the entropy generation rate ($\dot{S}_{gen}$). The simultaneous interaction of both PFHS resistance and viscous dissipation must be taken into account in the PFHS optimization procedure in order to establish optimal operating conditions. All variables of interest, namely number of fins (N), height of the fin (H), spacing between fins (b), and incoming air velocity (v_f), have been constrained between their lower and upper bounds, hence providing a simultaneous optimization of all design variables. The multi-objective optimization problem of the PFHS is solved using Non-dominated Sorting Genetic Algorithm-II (NSGA-II [10]). Two different configurations of the PFHS (a) PFHS with a flow through air cooling system and (b) PFHS with an impingement flow air cooling system have been considered in the present work. The formulations of these two configurations are adapted from Chen and Chen [5] and the function evaluations are set similar to them. The following parameters are used for all the optimization tasks in the present work:

- Population size = 100,
- Generations = 100,
- Crossover probability (Simulated Binary Crossover) = 0.9,
- SBX index = 10,

- Mutation (Polynomial mutation) probability = 1/number of variables,
- Mutation index = 100.

The Pareto-optimal solutions obtained from NSGA-II are shown in Fig. 3. Fig. 3 consists of the comparative performance of our method with Chen and Chen [5] for a PFHS with a flow through air cooling system. The non-dominated solutions in the figure clearly show that the performance of the present method is better than results by Chen and Chen [5].

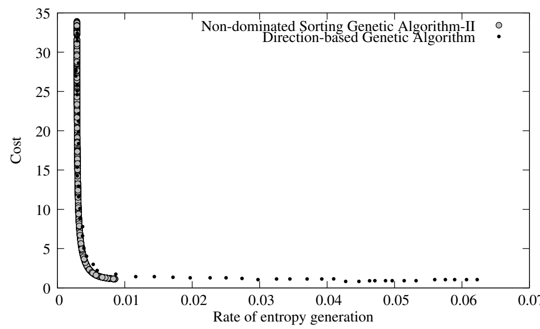


Fig. 3. Non-dominated solutions between rate of entropy generation and cost for flow-through

The entropy generation rate varies between $0.002898W/k$ and $0.008558W/k$. The lowest cost is $1.132713NTD$ whereas the highest cost is $33.920260NTD$. Table 1 shows two extreme values and some intermediate values of the objective function along with corresponding design variables. It can also be observed from Table 1 the number of fins is directly proportional to the rate of entropy generation. The number of fins attains its highest value (40) when the entropy generation rate ranges from $0.004296W/k$ to $0.008558W/k$. The incoming velocity achieves its highest value ($2m/s$) when entropy generation rate reaches $0.003149W/k$. We can observe that with further increase, the number of fins and incoming air velocity entropy generation will increase.

The resulting non-dominated solutions between rate of entropy generation and cost for the PFHS with an impingement flow air cooling system are shown in Fig. 4. The entropy generation rate ranges from $0.005247W/k$ to $0.008879W/k$ and the cost varies between $1.132710NTD$ and $2.771404NTD$. The extreme and intermediate values of objective functions for this configuration along with design variables are shown in Table 2. The table clearly shows that number of fins always takes its highest bound (40) and height of the fin is always $0.025mm$. The incoming air velocity is fixed at its upper bound i.e. $2m/s$. Table 2 also shows that three variables out of four are fixed for all non-dominated solutions. The only variable that causes different non-dominated solutions is spacing between the fins.

In the present work, in addition to solving the above mentioned two configurations, knowledge extraction has been carried out from the obtained solutions of multi-objective optimization. The motivation of the knowledge extraction is to establish a

Table 1. Non-dominated solutions along with variable values for PFHS with flow through

Rate of entropy generation (W/k)	Cost (NTD)	Number of fins	Height of the fin (mm)	Spacing between fins (mm)	Incoming air velocity (m/s)
0.002898	33.920260	18	0.124923	0.001144	1.186749
0.002934	21.023040	22	0.089170	0.001130	1.507174
0.003002	14.888590	25	0.072592	0.001143	1.757355
0.003149	9.480572	29	0.055671	0.001135	2.0
0.003552	5.350479	36	0.043285	0.001077	2.0
0.004296	2.956553	40	0.032595	0.001084	2.0
0.005281	1.876195	40	0.027582	0.001169	2.0
0.006376	1.420160	40	0.025001	0.001214	1.999995
0.007512	1.227456	40	0.025	0.001238	2.0
0.008558	1.132713	40	0.025	0.001250	1.999499

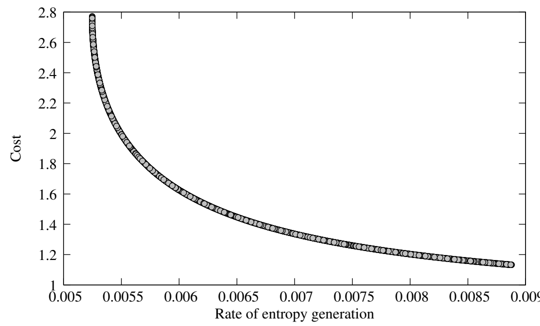


Fig. 4. Non-dominated solutions for rate of entropy generation and cost for impingement-flow

relationship between input design variables and output non-dominated solutions of the multi-objective optimization problem. This knowledge will help the user to select the fin for specific application.

4 Knowledge Extraction

The non-dominated solutions of multi-objective optimization were shown in the previous section. Next, the solutions are analyzed thoroughly to extract knowledge from the obtained non-dominated solutions. The motivation is to establish the existence of meaningful relationships between objective functions and decision variables. These relationships will help the decision maker select the appropriate configuration of the PFHS based on the specific and customized needs.

4.1 PFHS with flow through air cooling system

All the decision variables were plotted along with the first objective function (rate of entropy generation) to visualize the relationships between objective functions and design variables. As the two objective functions considered in this case are conflicting, it would

Table 2. Non-dominated solutions along with variables for PFHS with impingement flow.

Rate of entropy generation (W/k)	Cost (NTD)	Number of fins	Height of the fin (mm)	Spacing between fins (mm)	Incoming air velocity (m/s)
0.005247	2.771404	40	0.025	0.001042	2.0
0.005380	2.173199	40	0.025	0.001118	2.0
0.005551	1.935973	40	0.025	0.001148	2.0
0.005828	1.717386	40	0.025	0.001176	2.0
0.006183	1.548827	40	0.025	0.001197	2.0
0.006628	1.414482	40	0.025	0.001214	2.0
0.007165	1.308253	40	0.025	0.001228	2.0
0.007726	1.232049	40	0.025	0.001237	2.0
0.008302	1.175486	40	0.025	0.001245	2.0
0.008879	1.132710	40	0.025	0.001250	2.0

be sufficient to establish the relationship of the design variables with any one of the objective functions. The dependence on the other objective function with design variables can be predicted by exploiting the fact that both the objective functions are conflicting. However, it can be argued that analytical relationships between the objective functions and the design variables are obtained for both the objective functions separately so that the exact dependence on the design variables can be understood.

The change in the rate of entropy generation ($\dot{S}_{gen}$) with the variation in the number of fins (N) is shown in Fig. 5. Existence of two distinct zones is visible in Fig. 5. The first zone shows that as $\dot{S}_{gen}$ decreases, there is a corresponding decrease in number of fins. This plot also gives the exact relationship between $\dot{S}_{gen}$ and N in Zone 1 as shown in Eq. 5:

$$N = -1963.96 + (1.69185e^{6}\dot{S}_{gen}) - (4.79851e^{8}\dot{S}_{gen}^{2}) + (4.56218e^{10}\dot{S}_{gen}^{3}). \quad (5)$$

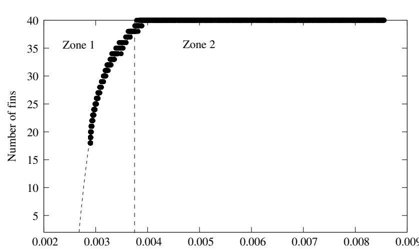


Fig. 5. Variation of number of fins with ($\dot{S}_{gen}$)

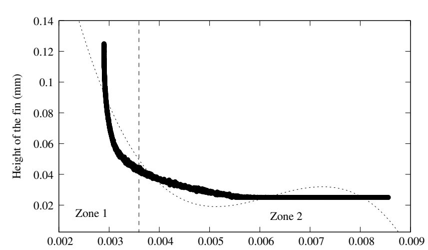


Fig. 6. Variation of height of fin with ($\dot{S}_{gen}$)

In Zone 2, the number of fin attains its maximum allowable value of 40 corresponding to a critical value of entropy generation rate, the value being 0.0039 w/k. Once this critical value is achieved, there is no change in the number of fins in the design with any further increment in the entropy generation rate. It should be noted that if any design calculation gives N as a non-integer value, it should be approximated with the nearest integer value. In the second zone, the number of fins is always fixed at its upper bound.

Fig. 6 shows variation in height of fin (H) with entropy generation rate ($\dot{S}_{gen}$) as obtained from the Pareto-optimal solutions of optimization results. The two zones do not have very clear distinction in Fig. 6. However, two different zones have been identified to have uniformity in the discussion. In the first zone, the variation in H with entropy generation rate, $\dot{S}_{gen}$, has a steep slope. However, the variation in H with respect to $\dot{S}_{gen}$, is very minimal in the second zone compared to the first zone. The relationship plot can be approximated by the cubic polynomial (Eq. 6):

$$H = 0.618 - 305.676\dot{S}_{gen} + 50881.1\dot{S}_{gen}^{2} - 2.741e + 06\dot{S}_{gen}^{3}. \tag{6}$$

Fig. 7 shows the relationship between variation of entropy generation rate ($\dot{S}_{gen}$) and spacing between the fins (b). The observation can be divided into two different zones. The first zone ranges from $0.002W/k$ to $0.0038W/k$ whereas the second zone lies from $0.0039W/k$ to $0.86W/k$. An inverse proportionality exists in the first zone. The variation in the first zone can be approximated using a linear equation of the form as given in Eq. 7 (Zone 1).

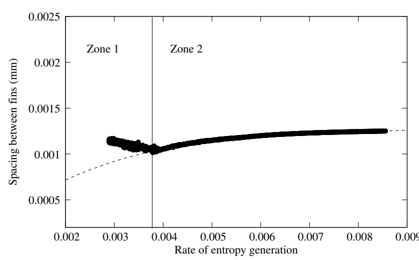


Fig. 7. Variation of space with ($\dot{S}_{gen}$)

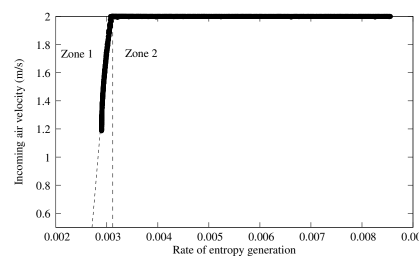


Fig. 8. Incoming air velocity with ($\dot{S}_{gen}$)

In the second zone, it is observed that $\dot{S}_{gen}$ bears higher order proportionality with b. The variation can be closely approximated with the help of a cubic polynomial of the following form (Eq. 7 (Zone 2)):

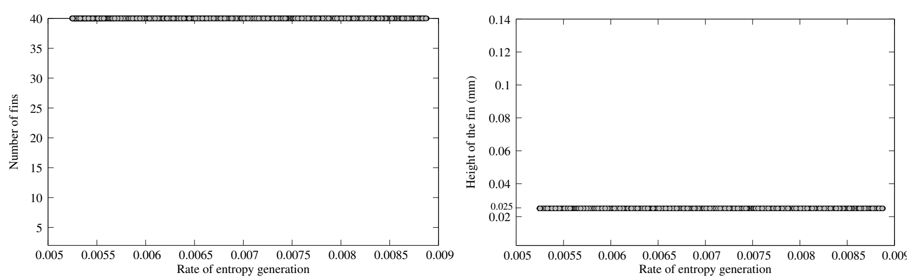
$$b = \begin{cases} 0.0015 - 0.1106\dot{S}_{gen}, & \text{Zone 1,} \\ 7.32236e - 05 + 0.41382\dot{S}_{gen} \\ -50.2806\dot{S}_{gen}^{2} + 2105.77\dot{S}_{gen}^{3}, & \text{Zone 2.} \end{cases} \tag{7}$$

The junction of Zone 1 and Zone 2 shows zeroth order continuity where values are continuous but the derivatives are discontinuous. Fig. 8 shows the variation of entropy generation rate ($\dot{S}_{gen}$) with incoming air velocity (v_f) as obtained from the result of post optimal analysis. The variation can be classified into two distinct zones. However, these two zones are dissimilar to the other three zones. In the first zone the entropy generation rate, $\dot{S}_{gen}$, varies linearly with incoming air velocity, v_f. This linear variation has a very high slope indicating that for a small change in entropy generation rate there is significant change in incoming air velocity. The linear variation ceases to exist at the critical value of entropy generation rate, which is $0.0033W/k$. The incoming air velocity attains its allowable maximum limit of $2m/s$ at the critical rate of entropy generation of $0.0033W/k$ and after that it remains unchanged with further variation in entropy generation rate in Zone 2. The analytical relationship between entropy generation rate ($\dot{S}_{gen}$) and incoming air velocity (v_f) in Zone 1 can be approximated as follows (Eq. 8):

$$v_f = -11.076 + 4269.79\dot{S}_{gen} \tag{8}$$

4.2 PFHS with impingement flow air cooling system

In an interesting observation, the rate of entropy generation ($\dot{S}_{gen}$) varies only with fin spacing parameter (b) while being invariant with the other three design variables (Fig. 9 (a), (b), and Fig. 11). To analyze how one optimal solution differs from the other optimal solutions, all four design parameters have been plotted against rate of entropy generation, $\dot{S}_{gen}$ (Fig. 9 - 10).



(a) Variation of number of fins (b) Variation of height

Fig. 9. Variation of number of fins and height of fin with rate of entropy generation ($\dot{S}_{gen}$)

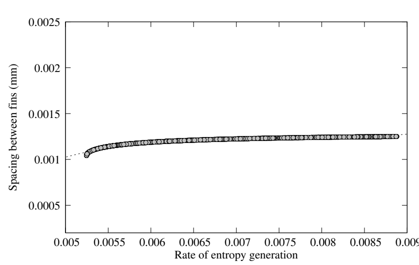


Fig. 10. Variation of space with ($\dot{S}_{gen}$)

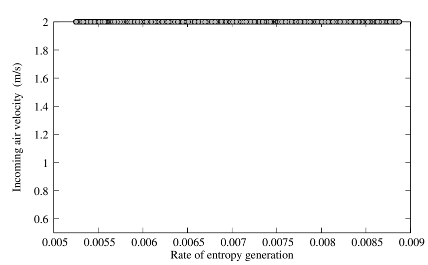


Fig. 11. Incoming air velocity with ($\dot{S}_{gen}$)

For a lower entropy generation rate, the fin spacing parameter should be small and with any increase in entropy generation rate, the fin spacing parameter increases monotonically with ($\dot{S}_{gen}$). This variation can be explained from the practical observation that for a fixed plate dimension a small fin spacing parameter would result in a larger number of fins. This increase in the number of fins would eventually result in a lower entropy generation rate. Therefore, if better fin performance is desired, it is advisable to design a PFHS with a lower value of fin spacing parameter and to keep all the other design variables at their prescribed constant values as discussed below. Hence, it can be inferred from this knowledge extraction methodology that if an optimal PFHS is to be designed with four design parameters (N, H, b, and v_f), the three parameters namely N, H, and v_f must be fixed whereas b can be adjusted to obtain the desired trade off among various chosen objectives. The relationship between the rate of entropy generation ($\dot{S}_{gen}$) and b from Fig. 10 is shown below (Eq. 9):

$$b = -0.0002 + 0.367\dot{S}_{gen} - 23.4294\dot{S}_{gen}^2 \tag{9}$$

Therefore, knowledge extraction not only is useful for design of optimal PFHS, but also allows the designer to make some well informed predictions about the behavior of the system with any possible change in design.

5 Conclusion

The optimization of PFHSs plays a meaningful role in the efficient resource utilization for a given cooling objective. A multi-objective evolutionary algorithm is applied to solve the optimization problem due to the existence of non-linear constraints and objective functions. NSGA-II is used due to its potential to deal with non-linear constraints and objective functions in multi-objective optimization problems. It is evident from non-dominated solutions that NSGA-II has successfully generated well-spread non-dominated solutions. The non-dominated solutions of a PFHS with a flow-through air cooling system are compared with the results obtained by a multi-objective real-coded genetic algorithm using a direction-based crossover operator by Chen and Chen [5], and it is shown that the NSGA-II results outperform the other method. The non-dominated solutions of both cases are analyzed to obtain the interrelationship that may exist among the variables and objective functions. The knowledge extraction results showed that the relationship is simpler in the case of the PFHS with an impingement-flow air cooling system compared to the PFHS with a flow-through air cooling system. These relationships can provide a deep insight to the users and designers.

The present study took into consideration four variables (number of fins, height of fin, space between number of fins, and incoming air velocity) in the optimization study. Another direction of research could be to increase the number of constraints and objective functions and solve for both configurations as many objective optimization problems. A generalized formulation for the above two configurations can be designed which can assist designers in the development of a PFHS used for cooling electronic devices based on their specific needs in terms of cooling rate, space availability, and material cost. The PFHSs can also be integrated with smart materials to introduce adaptability in their geometry and performance. This would enable fins to vary their geometry and heat flux rate in response to change in the value of the thermal parameters of the surrounding cooling medium. The present study can further be used to find the right combination of conventional and smart materials to yield the optimal value of thermal performance, material cost, and operational cost.

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